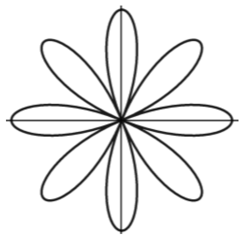
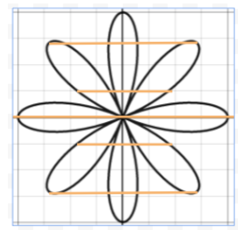
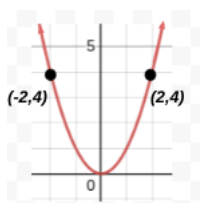
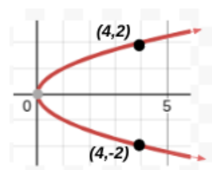
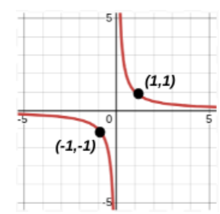
Math 1320: Tests for Symmetry

What is symmetry? Consider the flower shape below:





The flower is balanced and looks the same on all sides. On the right we have the same shape, but there are lines connecting the tips of 'petals' that are across from each other. Notice how the petals are perfectly aligned. The left side is a mirror image of the right and vice versa. The flower is symmetric!
When looking for symmetry in graphs of equations, we are checking that the features of one part of the graph are the reflection (or mirror) of another. There are three forms of symmetry:

Symmetric with respect to the **y-axis**	Symmetric with respect to the **x-axis**	Symmetric with respect to the **origin**
 If I folded my paper on the y-axis, the two halves of the graph match up.	 If I folded my paper on the x-axis, the two halves of the graph match up.	 If I spun the graph around the origin the graph would stay the same.
If for every point (x, y) on the graph, the point **(-x,y)** is also on the graph	If for every point (x, y) on the graph, the point **(x,-y)** is also on the graph	If for every point (x, y) on the graph, the point **(-x,-y)** is also on the graph
Test: Substituting $-x$ for x in the equation results in the same equation	**Test:** Substituting $-y$ for y in the equation results in the same equation	**Test:** Substituting $-x$ for x and $-y$ for y in the equation results in the same equation

The flower shape above is symmetric with respect to both the x- and y-axis. Similarly, equations may have more than one form of symmetry. Because of this, we will have to test for all three forms of symmetry using the tests from the table above.

⋆ Note: An equation that is symmetric with respect to the x-axis is not a function (since for every input there is more than one output).

Why is graph symmetry important? In this course, we will learn about even and odd functions, which is determined by the symmetry of a function. When we begin graphing equations by hand, using properties of even and odd functions will help us identify important behaviors of the graph and minimize the number of points we will need to plot.

What questions may I be asked about the symmetry of an equation's graph? You may be asked to identify the type of symmetry from a graph or equation. And, eventually, identify even and odd functions based on the symmetry of their graph or equation. For now, we will focus on identifying different forms of symmetry of a given equation.

Example 1. Determine whether the graph of $y = x^3 + 1$ is symmetric with respect to the y-axis, x-axis, or the origin.

1. Test for symmetry with respect to the y-axis. We must replace x with $-x$ and see if the result is the same as the original equation:

$y = x^3 + 1$	Given equation
$y = (-x)^3 + 1$	Replace x with $-x$
$y = -x^3 + 1$	Since $(-x)^3 = (-x)(-x)(-x) = -x^3$

   Because we do not get the same equation, the graph of $y = x^3 + 1$ is not symmetric with respect to the y-axis.

2. Test for symmetry with respect to the x-axis. We must replace y with $-y$ and see if the result is the same as the original equation:

$y = x^3 + 1$	Given equation
$(-y) = x^3 + 1$	Replace y with $-y$
$y = -(x^3 + 1)$	Multiply both sides by -1

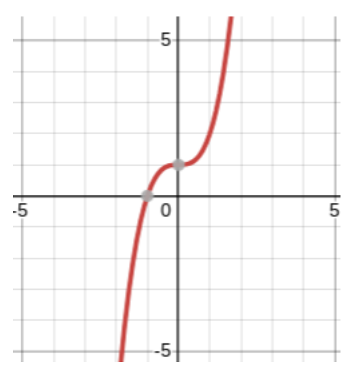
   Because we do not get the same equation, the graph of $y = x^3 + 1$ is not symmetric with respect to the x-axis.

3. Test for symmetry with respect to the origin. We must replace x with $-x$, y with $-y$ and see if the result is the same as the original equation:

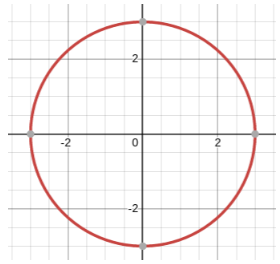
$y = x^3 + 1$	Given equation
$(-y) = (-x)^3 + 1$	Replace x with $-x$ and y with $-y$
$-y = -x^3 + 1$	Since $(-x)^3 = (-x)(-1)(-x) = -x^3$
$y = x^3 - 1$	Multiply both sides by -1

   We almost get the same equation as a result, but in the original equation we have $+1$ and in our new equation we get -1. Therefore, since we do not get the same equation, the graph of $y = x^3 + 1$ is not symmetric with respect to the origin.

4. This is an example of an equation whose graph has no form of symmetry. If we check the graph, we can see why there is no symmetry:



Example 2. Consider the equation $x^2 + y^2 = 9$. This is an equation of a circle, whose graph is below:



See how the graph of $x^2 + y^2 = 9$ is symmetric with respect to the y-axis, x-axis, and the origin! Use the tests for symmetry to confirm these three forms of symmetry.